
Assignment 04

1) MAP (No Matlab)

The time difference (in weeks) between occurrences of Zika virus symptoms may generally be estimated by the geometric distribution:

$$p(x|k) = \begin{cases} (1.85 - k)^{x-1} * k & x \in \{1,2,3, \dots\} \\ 0 & x \leq 0 \end{cases}$$

The parameter k describing the distribution is different for each patient ($0 < k < 1.85$). The five observations of amnesia in patient X are spaced by **2, 2, 5, 6, 8** weeks. Assuming the prior distribution $\mathbf{p}_0(\mathbf{k}) = \mathbf{e}^{-0.75\mathbf{k}}$ known for the general patient population.

a) What is the maximum a-posteriori (MAP) estimator for k ?

b) Assuming there was a 6th observation of **37** for patient X , is the MAP estimator affected? If so, calculate the new estimator.

c) Assuming a different prior distribution, defined as $\mathbf{p}_0(\mathbf{k}) = \begin{cases} 1/1.85, & 0 < \mathbf{k} < 1.85 \\ 0, & \text{otherwise} \end{cases}$,

How would the estimator be affected? (Consider the original 5 observations only)

d) What is the maximum likelihood (MLE) estimator for k ? Explain.

2) Population code (Yes Matlab)

Simulate the responses of three interneurons in the positioning system of an imaginary insect, check the accuracy of a vector decoding scheme:

For a true position θ , the average firing rates of the three interneurons is generated as

$$r_i = [67(\text{Hz}) * \cos(\theta - \theta_i)]_+$$

where $[\]_+$ indicates half-wave rectification (i.e. you take only the positive values, negative values are set to 0), and $\theta_i = 0, \pi/2, 1.3*\pi$ for $i = 1, 2, 3$ accordingly.

a) For each neuron, plot r_i as a function of θ . (all on the same plot)

b) Plot $\hat{\theta}$ (decoded by population vector) as a function of the original θ for the entire range ($0 \leq \theta < 2\pi$). Show the diagonal (perfect decoding) for comparison.

c) Which values of θ can be decoded **exactly** correctly? (Consider $0 \leq \theta < 2\pi$).

d) Which angles (or range of angles) can be uniquely decoded, and which cannot? Explain and show your answer graphically using b) (consider $0 \leq \theta < 2\pi$).

e) Bonus (15 pts.) - Error analysis (Consider the range $-3/4*\pi \leq \theta \leq 3*/4\pi$ only):

Introduce an independent additive Gaussian noise component, with zero mean and STD of **13Hz** to r_i (i.e. $r_{i_noisy} = [r_i + \text{noise}]^+$, so if $r_{i_noisy} < 0$, then $r_{i_noisy} = 0$). Repeat **100** times (trials) for each neurons (so for each neuron and each θ , 100 r_i noise values are recorded).

- I) Plot the average $\hat{\theta}$ (averaged decoded angle over trials) as a function of the original θ for the entire range (range $-3/4*\pi \leq \theta \leq 3/4*\pi$). Show the diagonal (perfect decoding) for comparison.
- II) Plot the RMS error (also called RMSD) of the population vector decoder as a function of θ . How does the noise affect the decoding performance over the θ range? Explain.