SIGNAL & DATA ANALYSIS IN NEUROSCIENCE 2018

OPTIMIZATION

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Optimization

Optimization - process of finding the conditions that give the maximum or minimum of a function.

Mathematical optimization problem:

minimize $f_0(x)$
subject to $g_i(x) \le b_i, \quad i = 1, ..., m$

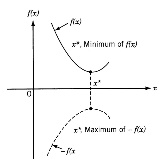


Figure 1.1 Minimum of $f(x)$ is same as maximum of $-f(x)$.

- f_0 : $\boldsymbol{R^n} \rightarrow \boldsymbol{R}$: objective function
- $x=(x_1, ...,x_n)$: design variables (unknowns of the problem)
- g_i : $\boldsymbol{R^n} \rightarrow \boldsymbol{R}$: $(i=1,...,m)$: inequality constraints

Global vs. Local

- A *global optimum* represents the very best solution while a *local optimum* is better than its immediate neighbors. Cases that include local optima are called *multimodal*.
- Generally desire to find the global optimum.

4

Solving an optimization problem

- Find optimal values x^* for the variables.
- Analytical methods make use of differential calculus in locating the optimum solution. These methods assume that the function is differentiable twice with respect to the design variables and the derivatives are continuous.
- What if the function is too complicated to find an analytical solution for the minimum? (Usually impossible to solve analytically)
- Must be solved numerically → approximation of the solution

5

Line search methods

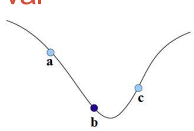
- optimize a given function with respect to a **single variable**
- Assume that f is unimodal in $[a, b]$, such that the minimum x* lies inside.
- General idea is to start reducing the interval $[a, b]$ s.t. the minimizer is still included in it
- An approximation of the minimizer is found when the length of the interval is smaller than a pre-determined tolerance
- $x^* \in [a^*, b^*]$ where $b^* - a^* <$ epsilon

- Differentiability is not essential.

6

Initial bracketing interval

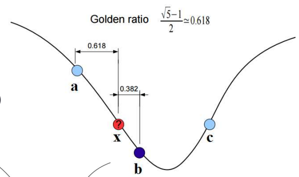
Consider two points
- a < b
- f(a) > f(b)



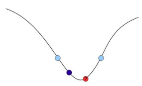
- Take successively larger steps beyond b until function starts increasing
- Results in 3 points: a, b, c, such that
  - a < b < c
  - f(b) < f(a) and f(b) < f(c)

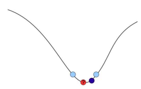
Golden search algorithm

- If [a,b] < [b,c] then
  set x = b + 0.38197 ·(c-b)
- Else
  set x = b - 0.38197 ·(b-a)



- Redefine a, b, c, such that
  - a < b< c
  - f(b) < f(a) and f(b) < f(c)





- Iterate this golden sectioning until |c-a| < ε