SIGNAL & DATA ANALYSIS IN NEUROSCIENCE 2020

SPECTRAL ANALYSIS

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Outline

- Power spectral density (PSD)
- Cross spectral density (CSD)
- Coherence

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Spectral analysis

- Parseval's theorem:

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df \quad , \sum_{n=0}^{N-1} |x[n]|^2 = \sum_{k=0}^{N-1} |X[k]|^2$$

- Power vs. energy: Power = Energy / Time
- Wiener-Khinchin theorem: The power spectrum is the Fourier transform of the auto-correlation function.

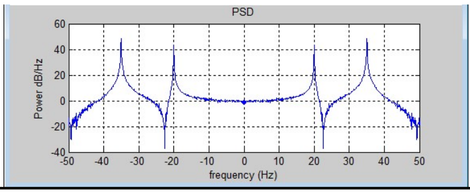
$$S(f) = \int_{-\infty}^{\infty} R(\tau)\, e^{-2\pi i f \tau}\, d\tau = \mathcal{F}(R(\tau)).$$

- Power spectral density (PSD)

$$P_{x,x}(f) = \frac{S_{x,x}(f)}{f_s}$$

Example PSD

```
Fs = 100;        t = 0:1/Fs:10;
x = 100*cos(2*pi* 20*t)+ 200*cos(2*pi* 35*t)+ randn(size(t));
Pxx = abs(fftshift(fft(x))).^2/(length(x)*Fs);
freq = -Fs/2 : Fs/(length(Pxx)-1) : Fs/2;
plot(freq, 10*log10(Pxx));
xlabel('frequency (Hz)'); ylabel('Power dB/Hz'); title('PSD');
```



PSD using Matlab's pwelch

[Pxx,F] = pwelch(x,window,noverlap,nfft,fs);

- Input parameters:
  - x – the signal to be processed
  - Window – Window or window length. Specifying a number uses a Hamming window of the specified length (e.g. 1000 bins, a 1 second window)
  - noverlap – Overlap (in time) between calculation windows - how many time bins fall in both neighboring windows (e.g. 500 bins, 50% overlap)
  - NFFT – How many "points" in the resulting DFT.
    Can set to window length, works faster with lengths that are powers of 2:
    nfft = 2^(nextpow2('window length')).
  - Fs – Sampling frequency
- Output parameters:
  - F – the array of estimated frequencies, F(2) is the resolution = 1/(Window length in seconds)
  - Pxx – PSD estimation at the values of F

Pwelch cont.

Example 1: PSD with no overlap, bin of 1Hz:
  pwelch(x, fs, 0, fs, fs);

Example2: PSD with 50% overlap, bin of 0.5Hz:
  pwelch(x, 2*fs, fs, 2*fs, fs);

Example3: PSD 50% overlap, bin of 2Hz:
  pwelch(x, fs/2, fs/4, fs/2,fs);

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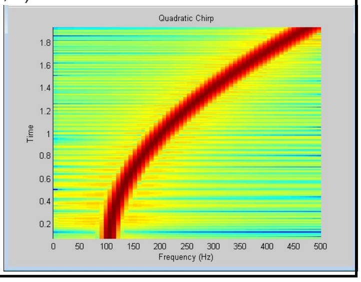
Spectrogram

```
T = 0:0.001:2;
X = chirp(T,100,1,200,'q');
spectrogram(X,128,120,128,1000);
  %(x,window,noverlap,nfft,fs)
title('Quadratic Chirp');
```



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Exam 2005A: power spectrum

- The EEG of the Golden Unicorn is made up of three frequencies 7, 18 & 100Hz. The amplitude of each component (frequency) is 10μV. The waveform is sampled at 150 samples/sec for 1 second. Sketch and explain the power spectrum of the EEG. What are the X & Y axes? What is the X scale resolution? What is the maximal frequency?

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Cross spectrum

- Multiplication of the Fourier transform of one signal with the conjugate of the other signals.

$$\hat{S}_{y,x}(\omega) = \frac{1}{N} Y(\omega) X^*(\omega)$$

- The transform of the cross correlation

$$S_{x,y}(\omega) = \sum_{-\infty}^{\infty} R_{x,y}(m) \cdot e^{-j\omega m}$$

Coherence

$$C_{xy} = \frac{|S_{xy}|^2}{S_{xx}S_{yy}}$$

$$0 \leq C_{xy} \leq 1$$

$$S \quad \text{spectral density}$$

- Normalizes to the spectrum of the two signals and thus relates only to the relation between the signals and not to their structure.
- Points on spectral linear correlations.

Example

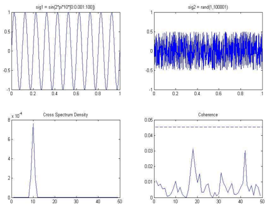
```
sig1 = sin(2*pi*10*[0:0.001:100]);

sig2 = rand(1,100001);
sig2 = sig2-mean(sig2);

figure;
subplot(2,2,1)
plot([0:0.001:1],sig1(1:1001))
title('sig1 = sin(2*pi*10*[0:0.001:100])')
subplot(2,2,2)
plot([0:0.001:1],sig2(1:1001))
title('sig2 = rand(1,100001)')

[Pxy,F1] = cpsd(sig1,sig2,1000,0,1000,1000);
subplot(2,2,3)
plot(F1(2:50),abs(Pxy(2:50)))
title('Cross Spectrum Density')

[Cxy,F2] = mscohere(sig1,sig2,1000,0,1000,1000);
subplot(2,2,4)
plot(F2(2:50),abs(Cxy(2:50)))
title('Coherence')
```



Example

```
t = [0:0.001:100];
alpha = pi; beta = pi*1.5;
f1 = 20;
freqInds = 2:50;
pLims = [0 0.5];
sig1 = sin(2*pi*f1*t +alpha);
sig2 = sin(2*pi*f1*t+beta);
figure; subplot(2,2,1);
plot([0:0.001:0.25],sig1(1:251));
title(['sig1 = sin(2*pi*' num2str(f1) '*t)'])
subplot(2,2,2)
plot([0:0.001:0.25],sig2(1:251));
title(['sig2 = sin(2*pi*f1*t+beta)'])
[Pxy,F1] = cpsd(sig1,sig2,1000,0,1000,1000);
subplot(2,2,3)
plot(F1((freqInds)),abs(Pxy((freqInds)))); ylim(pLims);
title('Cross Spectrum Density')
[Cxy,F2] = mscohere(sig1,sig2,1000,0,1000,1000);
subplot(2,2,4)
plot(F2((freqInds)),Cxy((freqInds)));ylim([0 1.2]);
title('Coherence (mscohere)')
```