SIGNAL & DATA ANALYSIS IN NEUROSCIENCE 2020

DISCRIMINATION

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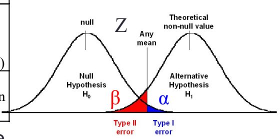
Discrimination: basic terms

- Problem: upon receiving 'r' decide upon H_0 or H_1.
- Decision rule: if $r \geq Z$ then decide H_1, otherwise decide H_0.

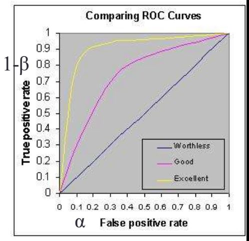
		Actual value: P	Actual value: N
Prediction	P	* True P * Detection * 1-β(z)	* False P * False detection /False alarm * Type I error α(z)
Prediction	N	* False negative * Miss detection * Type II error β(z)	* True N * Correct rejection * 1- α(z)



- $\alpha(Z) = P(r \geq Z \mid H_0)$ false positive.
- $\beta(Z) = P(r < Z \mid H_1)$ false negative.
- Sensitivity = 1 - β(Z). Sensitivity = 1 => recognize all positives.
- Specificity = 1 - α(Z). Specificity =1 => recognize all negatives.
- Power of the test 1-β.

Receiver Operating Characteristics (ROC) curve

- ROC generation:
  - for every threshold z extract FP & TP.
- Classification performance:
  - $\int(1-\beta)\, d\alpha$:
    ½ = worthless
    1 = perfect classification



- It may make sense to set α high to minimize Type II errors (false negatives), even at the expense of additional Type I errors (false positives). Example: radar, new drug test.

Example: ROC from exam 2007

Given the following probabilities of evoked potential amplitudes:

Neuron rate (spikes/sec)	0	10	20	30	40
Prey	0	0.2	0.2	0.3	0.3
Predator	0.3	0.3	0.2	0.2	0

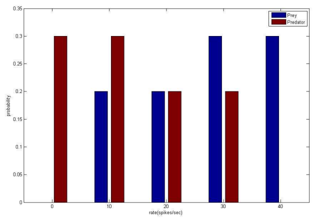
The behavior of the animal may be characterized by:

$$P(TP) = \sqrt{P(FP)}$$

- Plot the ROC curves of the behavior and neuronal discrimination.
- Is the single neuron sufficient to predict the behavior?
- What is p[success] for the two statistics? (reminder: P(success) = ∫(1-β) dα)

Solution

Rate spike/sec	0	10	20	30	40
Predator H_0	0.3	0.3	0.2	0.2	0
Prey H_1	0	0.2	0.2	0.3	0.3



H_0 – predator, **H_1 – prey**

Solution cont.

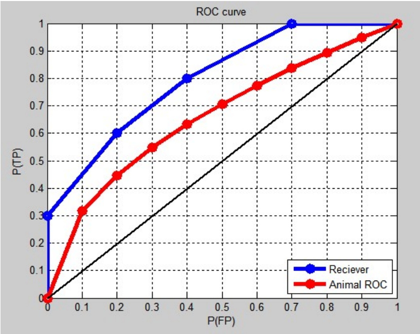
For each value of Z extract:
X axis: FP(Z) = P(rate ≥ Z $|H_0$).
Y axis: TP(Z) = 1 -P(rate < Z $|H_1$)

Z		-10	0	10	20	30	40	50
FP(Z)	= P(rate ≥ Z $|H_0$)	1	1	0.7	0.4	0.2	0	0
TP(Z)	= 1 -P(rate < Z $|H_1$) = P(rate ≥ Z$|H_1$)	1	1	1	.8	.6	.3	0

Animal ROC: $TP = \sqrt{FP}$

FP	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
TP = √FP	0	.3	.45	.55	.65	.70	.80	.85	.90	.95	1

Solution cont:



Solution cont

- Is the single neuron sufficient to predict the behavior? Ans: Yes, since the ROC provides performance that is above the behavior ROC, a single neuron is sufficient to predict behavior.

- P(success)

$$\frac{(.3+.6)\cdot .2}{2}+\frac{(.6+.8)\cdot .2}{2}+\frac{(.8+1)\cdot .3}{2}+\frac{(1+1)\cdot .3}{2}=.8$$

- P(success animal)

$$\int_0^1 x^{\frac{1}{2}}\cdot dx=\frac{2}{3}\cdot x^{\frac{3}{2}}\Big|_0^1=\frac{2}{3}$$

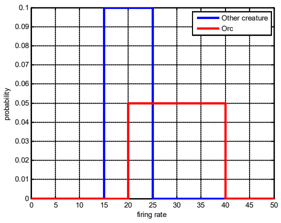
Continuous ROC (exam 2010)

- Bilbo Baggins has a neuron which is part of the Orc sensing system. Upon sensing an Orc the neuron's firing rate is described by a uniform distribution in the range [20-40] while upon sensing any other creature the firing rate is taken from a uniform distribution in the range [15-25].

a) **Draw** the ROC curve of the Orc identification.

b) **Calculate** the classification performance.

ROC exam 2010 - Solution



- Z = 10 : p(FP) = 1, p(TP) = 1
- Z = 15 : p(FP) = 1, p(TP) = 1
- Z = 20 : p(FP) = 0.5, p(TP) = 1
- Z = 25 : p(FP) = 0, p(TP) = 0.75
- Z = 40 : p(FP) = 0, p(TP) = 0

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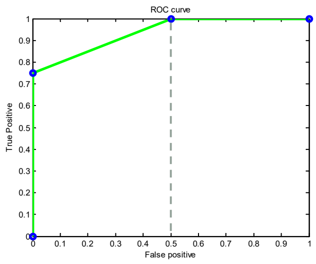

- Classification performance :

$\int TP\, dFP = [1*0.5 + (0.75+1)/2 * 0.5] = 0.9375$