SIGNAL & DATA ANALYSIS IN NEUROSCIENCE 2020

POISSON POINT PROCESS

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Poisson point process

- The probability of an event to happen is independent of previous events.
- It is totally dependent on the p.d.f that govern the process.
- $P(t_n|t_1,\ldots,t_{n-1})=P(t_n)$
- Homogeneous Poisson process: Constant firing rate *r*.
- Inhomogeneous Poisson process: Time dependent firing rate *r(t)*.

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Homogeneous Poisson process

- The **mean** firing rate of the neuron is constant $r(t) = r$
- Every sequence of *n* spikes over a fixed time interval *T* has an equal probability. But not equal probability across different *n*'s !
- We call this probability $P_T[n]$ – the probability of getting *n* spikes in time span *T*.

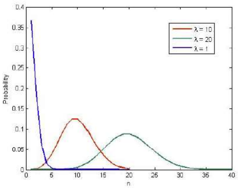
$$P_T[n] = \frac{(rT)^n}{n!} e^{-rT}$$

Poisson distribution

$$\lambda = r \cdot T \qquad P(n) = \frac{\lambda^n}{n!} e^{-\lambda}$$

- λ is the average events *n* in the interval *T*
- Mean = Variance = *rT* = λ

MATLAB - poisspdf(a, lambda)
poisspdf(1:20, 10)



Terms: definitions and properties

Term	Definition
Inter-spike interval (ISI)	Homogeneous Poisson process: Discrete: P(ISI = k intervals) = $(1-p)^{k-1} \cdot p$ Continuous: $P(N(t, t+\tau) = k) = \frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!}$ P(ISI=τ) = P(N(0, τ-dt) =0) · P(N(τ-dt, τ) =1)
Coefficient of variation (Cv)	$C_v = \frac{\text{std (ISI)}}{\text{E(ISI)}}$
Fano factor (FF)	$\text{FF} = \frac{\text{var(spike count over T)}}{\text{E(spike count over T)}}$

Properties of Poisson distribution

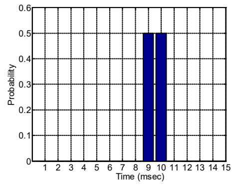
- For a homogeneous Poisson process
  $\mu = \sigma^2 = rT = \lambda$
- For a homogeneous Poisson process
  FF = Cv = 1.

  FF=1 & Cv=1 is a necessary, though not sufficient, condition to identify a Poisson spike train.

- For a regular process FF = Cv = ?
- Cv >1 → higher order process.

Example: Exam 2007

- The ISI histogram of a neuron in the Quasis Regularlis ganglion is plotted below:



- Calculate the CV, compare to Poisson.
- Estimate the Fano factor (100msec window for rate calculation), compare to Poisson.

Solution

- Let us derive **Cv**:
  E[ISI] = 0.5*9 +0.5 *10 = 9.5
  var[ISI] = 0.5(9 -9.5)^2 + 0.5(10 -9.5)^2 =0.5^2

$$C_V = \frac{\sqrt{\mathrm{var[ISI]}}}{\mathrm{E[ISI]}} = \frac{\sqrt{0.5^2}}{9.5} << 1$$

  For a Poisson process Cv →1. In our case the process is close to regular.

- Let us estimate **FF** over T =100msec windows:
  E[#spike over T] = T/E[ISI] = 100 /9.5 ~10.5
  min [#spike over T] = 100/10 = 10.
  max [#spike over T] = 100/9 ~ 11.
  var[#spike over T] ~0.5^2 (We can estimate without exact values)

$$FF = \frac{var[\#spikes\ over\ T]}{E(\#\ spikes\ over\ T]} \approx \frac{0.5^2}{10.5} \ll 1$$

Example: Exam 2006

- Neurons in the Singelitis Non-Exactis nucleus of the Mountain Elf fires a spikes with intervals of either (p=0.5) 100ms or (p=0.5) 10ms.
  Calculate the Coefficient of variation of the neurons.
  Explain the results and their relation to a Poissonian neuron firing at the same rate.

Solution

- Cv definition: $C_v = \frac{\text{std (ISI)}}{\text{E(ISI)}}$

E(ISI) = 0.5 *100[msec] +0.5*10[msec] = 55[msec]
var(ISI) = 0.5*(100 -55)^2 +0.5*(10-55)^2 = 45^2

$$C_v = \frac{45}{55} = 0.81$$

- Note that Cv is high although the neuron is not Poisson.

Example: Exam 2005

- A pacemaker neuron fires 10 spikes/s in a fairly regular manner ISI=N(100ms,5ms) draw the ISI distribution and estimate the Fano factor and calculate the coefficient of variation.

Normal distribution: $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Solution

- Normal distribution: $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
- $C_v = \frac{\text{std (ISI)}}{\text{E(ISI)}} = \frac{5}{100}$
- Estimating **FF** over T = 100msec windows:
  E(#spikes): p(0 spikes) ~ p(1 spike) ~ 0.5
  Assuming 2 STDs (10 ms):
  min [#spike over T] = 100/110 ~ 0.
  max [#spike over T] = 100/90 ~ 1.
  var[#spike over T] ~0.5^2
  FF = 0.5

Generation of a Poisson process

- Common way: Translate the Poisson RV into a binomial RV in each bin, Simulate the probability for a spike using uniform distribution (MATLAB rand function).
- Using exponential distribution to evaluate ISI.