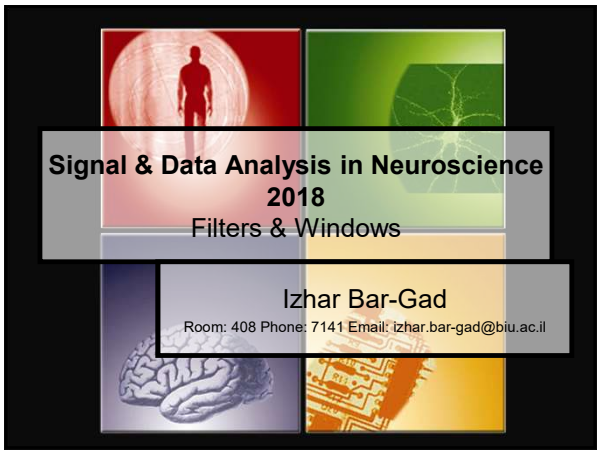

Outline – Frequency Domain

- ☑ Introduction
- ☑ Fourier Transform
- ☑ Sampling Theory
- ☑ Systems
- Filters
- Windows
- Spectral Analysis

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Filter in signal analysis

- Filters are used to **eliminate** unwanted frequencies from an input signal or to **select** a desired frequency range among many others.
- Filters form a sub-group of the systems.
- There are analog and digital filters. We will focus on the digital filters.

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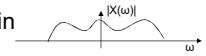
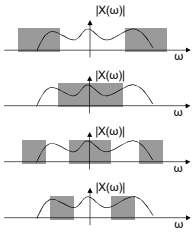
Filters

Original frequency domain



- Low Pass Filter (LPF)
- High Pass Filter (HPF)
- Band Pass Filter (BPF)
- Band Stop Filter (BSF)



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Input – Filter – Output

- Given the input X(ω) and desired output Y(ω), how can we describe H(ω) ?
- Moreover, how can we transfer it to the time domain and get a representation of the three components?

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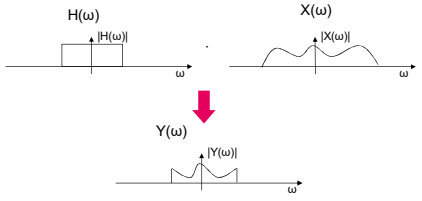
Ideal Filtering

Since $Y(\omega)=X(\omega)\cdot H(\omega)$



Remember : No frequency domain systems exist !!

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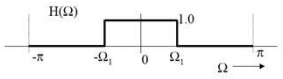
Ideals problem : F(rect)=sinc

Ideal frequency response → sinc in the time domain !

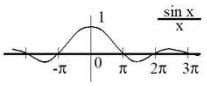
example:
ideal low-pass



$$h[n]=\frac{1}{2\pi}\int_{-\Omega_1}^{\Omega_1}\exp(j\Omega n)\,d\Omega=\frac{1}{2\pi}\left[\frac{\exp(j\Omega n)}{jn}\right]_{-\Omega_1}^{\Omega_1}$$

$$=\frac{1}{2\pi jn}\left[\exp(j\Omega_1 n)-\exp(-j\Omega_1 n)\right]$$

$$=\frac{1}{n\pi}\sin(n\Omega_1)=\frac{\Omega_1}{\pi}\operatorname{sinc}(n\Omega_1)$$



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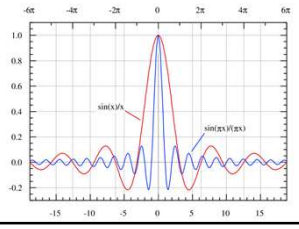
The sinc function

- The classical "unnormalized" definition $sinc(x)=\frac{\sin(x)}{x}$
- Signal analysis "normalized" definition $sinc(x)=\frac{\sin(\pi x)}{\pi x}$



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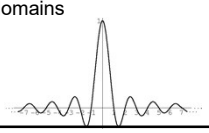
sinc problem

- The sinc function is infinite, forever oscillating around zero.
- Inserting Irrelevant samples into the filtered signal
- Since both F(rect)=sinc & F(sinc)=rect
  this problem is relevant for rectangular windows in both time and frequency domains



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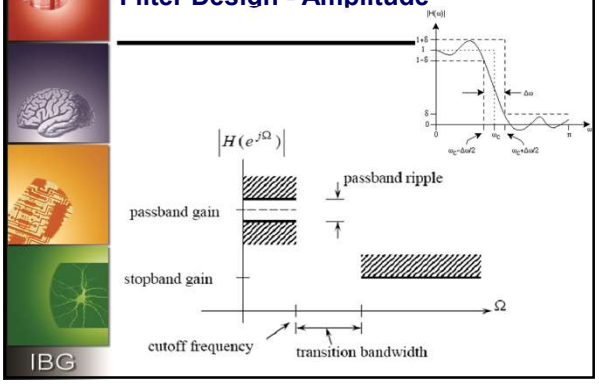
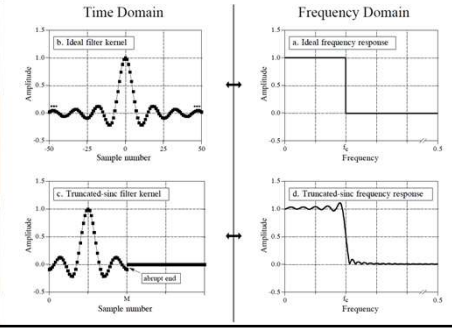
Filter Design - Amplitude

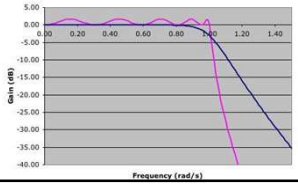


Truncated sinc filter



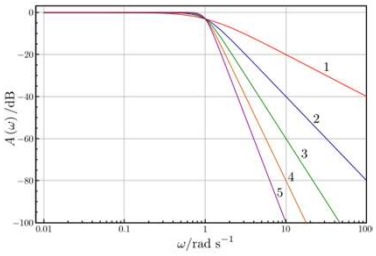
Filter families

- There are many filter families possessing different characteristics
- In this example a 10 pole Chebyshev & Butterworth low-pass filters



Number of poles

- Butterworth filter of order 1 → 5



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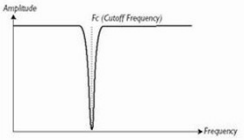
Special case: Notch Filter

- Special case of band-stop filter.
- Remove a narrow band from the signal
- Usually used for network noise (50,60 hz)
- BEWARE:
  - Drastically distorts phase, esp. when analog
  - Even if not, cancels most of the energy in the narrow band instead of just the noise part of it



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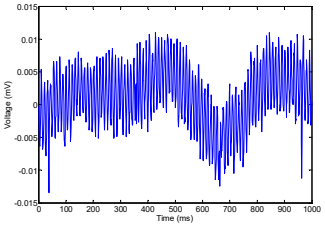
Filter example I

Original extracellular signal

Extracellular recording, containing the summation of different processes. Some relevant (neural origin) and some irrelevant (non neural origin).



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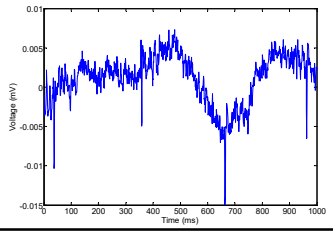
Filter example II

Following notch filter (~1800Hz)

External (non biological noise) may have a very narrow band. Both local field potential (LFP) and action potentials (spikes) are apparent.



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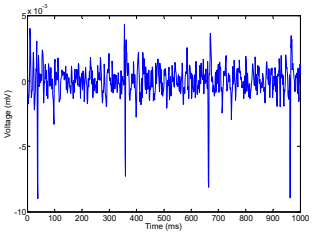
Filter example III

Following high pass (~300Hz)

Removal of the high frequency components removes the local field potential, leaving the spiking activity.



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Filter Design - Phase

LTI system: $H(\omega) = A(\omega)e^{i\theta(\omega)}$

$$P(\omega) \triangleq -\frac{\Theta(\omega)}{\omega}. \quad \text{(Phase Delay)}$$

Therefore, in order to achieve constant delay (that all frequencies will be delayed by the same phase) we would like to have a linear phase response.

[or at least at the relevant frequency range]

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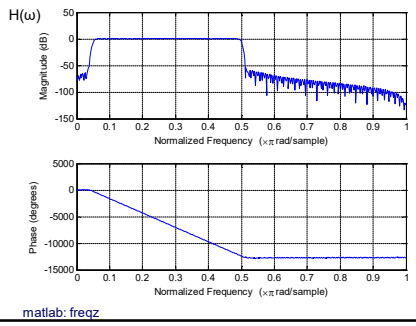
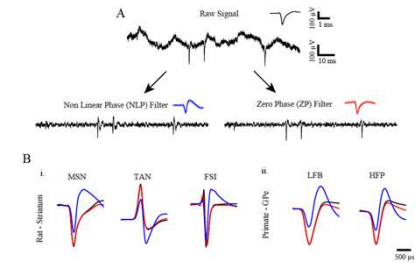
Semi-Log plot



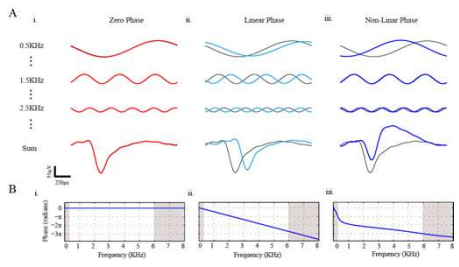
matlab: freqz

Phase distortion
Extracellular recording



(Yael & Bar-Gad., 2017)

Phase distortion
Linear vs. Non-linear phase



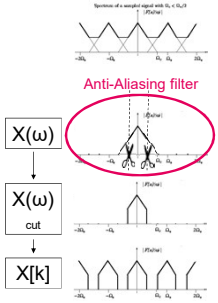
Filters
Avoiding sampling problems

when X(ω) → X[k] looks like this,

You have an aliasing problem…

What is there to do?

And if we can not increase ω_s ?



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Online vs. Offline Filtering

- Online filtering –
  - Live streaming (usually analog) data
  - Delay limit
  - Limited computational resources
- Offline filtering –
  - Sampled, stored, digital data
  - No time limit
  - Less limited computational resources

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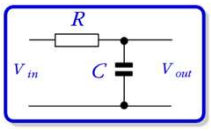
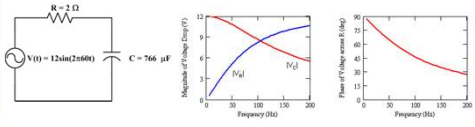
Analog Filters

RC Circuit = the simplest LPF





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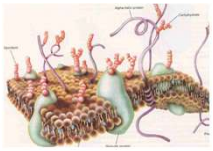
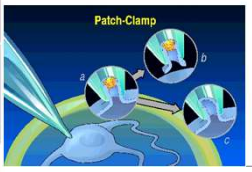
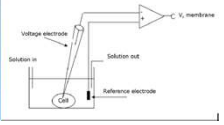
Filters in Neuroscience

- Cell Membrane as LPF
- Slice Bath as LPF







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Outline – Frequency Domain

- ☑ Introduction
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- Windows
- Spectral Analysis

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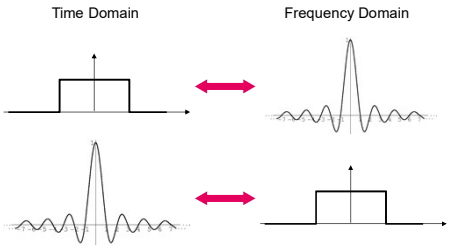
sinc: time & frequency duality



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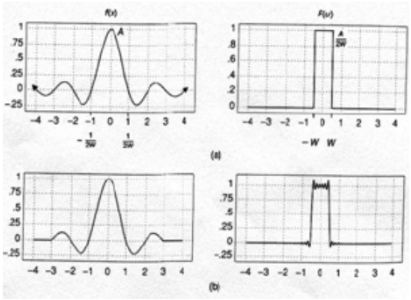
Finite sinc



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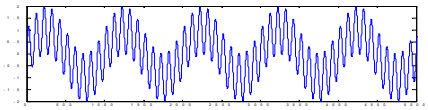
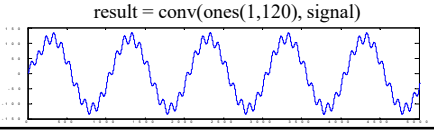
Ideal Filter Example

$$signal = \sin(\frac{2\pi}{1000}t) + \sin(10 * \frac{2\pi}{1000}t)$$



Convolution with a rectangular window

result = conv(ones(1,120), signal)



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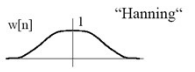
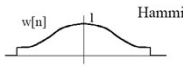
Windows

rectangular window - sharp edges - spreads out in frequency domain
we seek a more gentle window function, tapered towards the edges

$$w[n] = 0.5 + 0.5\cos\left(\frac{n\pi}{M+1}\right)$$
$$-M \le n \le M$$



$$w[n] = 0.54 + 0.46\cos\left(\frac{n\pi}{M}\right)$$
$$-M \le n \le M$$



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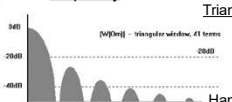
Windows frequency response

frequency time

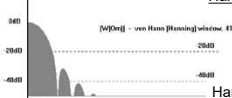
Triangular window



$$W[n] = 1 - \frac{2\left|n - \frac{M-1}{2}\right|}{M-1}$$

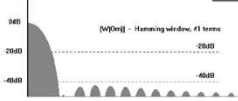
$-M \le n \le M$

Hanning window



$$w[n] = 0.5 + 0.5\cos\left(\frac{n\pi}{M+1}\right) \quad -M \le n \le M$$

Hamming window



$$w[n] = 0.54 + 0.46\cos\left(\frac{n\pi}{M}\right) \quad -M \le n \le M$$

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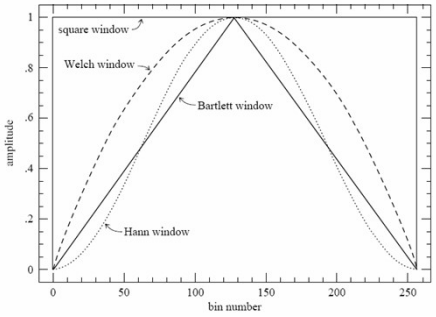
35

Leakage windows I



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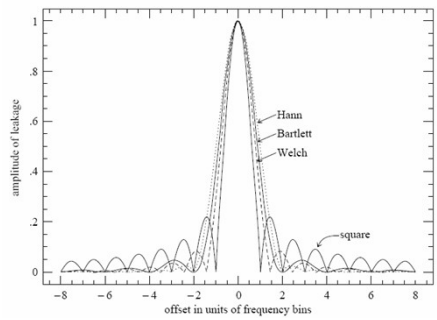
Leakage windows II



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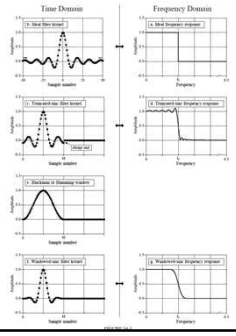
Windows and filters



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DFT Frequencies

- Discrete
- Range : 0-2π,
- Periodic

$$e^{-j\frac{2\pi k}{N}n} \rightarrow \Delta k = \frac{2\pi}{N}$$

But…

What about frequencies in between?

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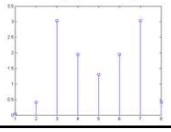
DFT Leakage

Say we had a sinusoid of 2.3 Khz and we still sampled it at 8000 samples/s :

The output frequency bins = 0Kkz, 1Khz, …, 7Khz
2.3 Khz bin is missing!!

What happens is that this frequency component shows up (leaks) in all the other frequency bins



Remedy
Windowing Techniques

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