Signal & Data Analysis in Neuroscience
Bar Ilan, Spring 2017 Due: 15th May 8:00 AM.

Assignment 05

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1) MAP (No Matlab)

The time difference (in weeks) between occurrences of Zika virus symptoms may generally be estimated by the geometric distribution:

$$p(x|k) = \begin{cases} (1.85-k)^{x-1} * k & x \in \{1,2,3,\dots\} \\ 0 & x \le 0 \end{cases}$$

The parameter k describing the distribution is different for each patient (0<k<1.85). The five observations of amnesia in patient X are spaced by **2, 3, 5, 4, 8** weeks. Assuming the prior distribution $\mathbf{p_0(k) = e^{-0.75k}}$ known for the general patient population.

a) What is the maximum a-posteriori (MAP) estimator for k?

b) Assuming there was a 6th observation of **37** for patient X, is the MAP estimator affected? If so, calculate the new estimator.

c) Assuming a different prior distribution, defined as $\mathbf{p_0(k)} = \begin{cases} \mathbf{1/1.85}, & \mathbf{0 < k < 1.85} \\ \mathbf{0}, & \mathbf{otherwise} \end{cases}$,

How would the estimator be affected? (Consider the original 5 observations only)

d) What is the maximum likelihood (MLE) estimator for k? Explain.

2) Population code (Matlab)

Simulate the responses of three interneurons in the positioning system of an imaginary insect, check the accuracy of a vector decoding scheme:

For a true position θ, the average firing rates of the three interneurons is generated as

$$r_i = [55(\text{Hz})*\cos(\theta - \theta_i)]+$$

where [ ]+ indicates half-wave rectification (i.e. you take only the positive values, negative values are set to 0), and θ_i = 0, 2π/3, 4π/3 for i= 1,2,3 accordingly.

a) For each neuron, plot r_i as a function of θ. (all on the same plot)

b) Plot $\hat{\theta}$ (decoded by population vector) as a function of the original θ for the entire range ($0 \le \theta < 2\pi$). Show the diagonal (perfect decoding) for comparison.

c) Which values of θ can be decoded correctly? (Consider $0 \le \theta < 2\pi$).

d) Bonus (10 pts.) - Error analysis (Consider the range $-\pi/4 \le \theta \le \pi/4$ only):

Introduce an independent additive Gaussian noise component, with zero mean and STD of **4Hz** to r_i (i.e. r_i_noisy = $[r_i + \text{noise}]^+$, so If r_i_noisy <0, then r_i_noisy = 0). Repeat **100** times (trials) for each neurons (so for each neuron and each θ, 100 r_i noise values are recorded).

I) Plot the average $\hat{\theta}$ (averaged decoded angle over trials) as a function of the original θ for the entire range (range π/4 ≤ θ ≤π/4). Show the diagonal (perfect decoding) for comparison.

II) Plot the RMS error (also called RMSD) of the population vector decoder as a function of θ. How does the noise affect the decoding performance over the θ range? Explain.

3) Optimization – Golden search(Matlab)

The file gs_data.mat contains a set of 20 observations drawn from a mixture of two normal distributions. Assuming that the two distributions have unit variance and symmetric means μ. And -μ., the likelihood function for these data is:

$$L(\mu) = \prod_{1} \frac{1}{\sqrt{2\pi}} \left[e^{-\frac{1}{2}(x_i - \mu)^2} + e^{-\frac{1}{2}(x_i + \mu)^2} \right]$$

(The product should be calculated over all observations)

a) Bracket the maximum of the function. What are your initial values of the points a,b,c ?

b) Using the golden-search optimization strategy find the MLE for μ.

c) How many function evaluations did you need for steps a and b?