SIGNAL & DATA ANALYSIS
IN NEUROSCIENCE

FREQUENCY DOMAIN PART-II

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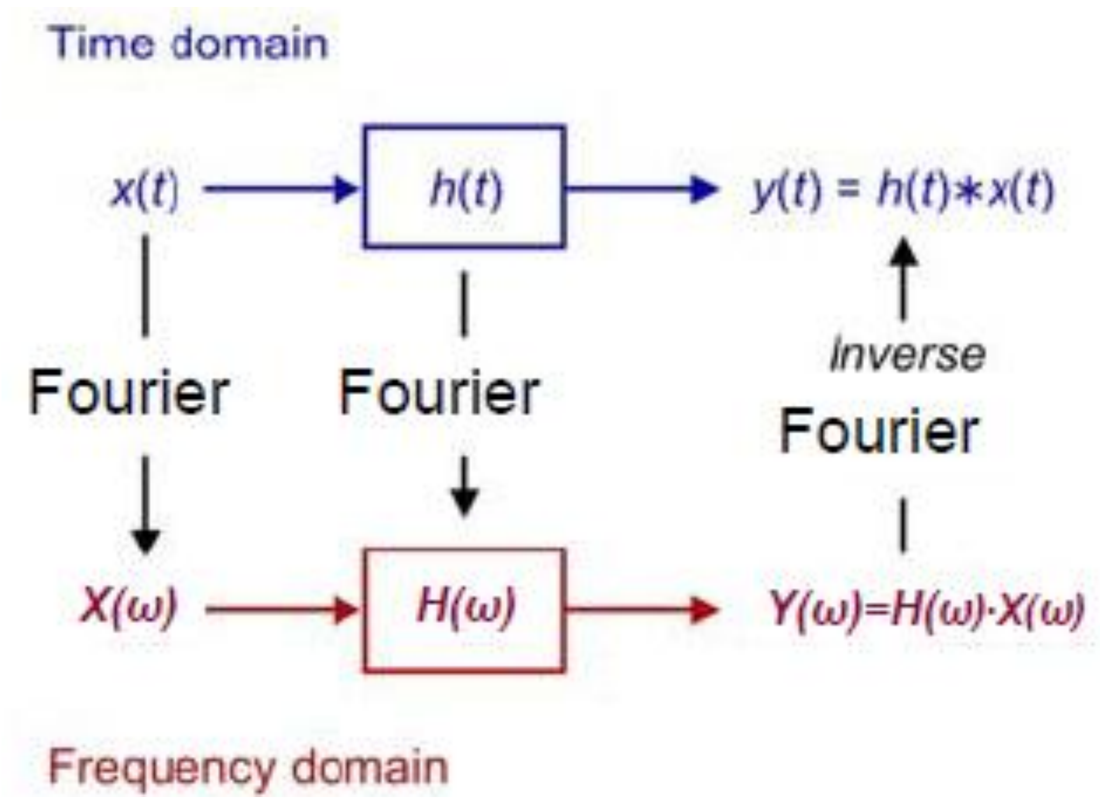
Agenda

- Spectral analysis
- Filters

Spectral analysis

Reminder: Time domain vs. frequency domain

- Convolution in the time domain is equivalent to multiplication in the frequency domain.



Spectral analysis

- Parseval's theorem:

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df \qquad , \sum_{n=0}^{N-1} |x[n]|^2 = \sum_{k=0}^{N-1} |X[k]|^2$$

- Power vs. energy: Power = Energy / Time
- Wiener-Khinchin theorem: The power spectrum density(PSD) is the Fourier transform of the auto-correlation function.

$$S(f) = \int_{-\infty}^{\infty} R(\tau)\, e^{-2\,\pi\, i\, f\, \tau}\, d\tau = \mathcal{F}(R(\tau)).$$

  Fourier transform of the autocorrelation
  is simply given by $|X(f)|^2$.
  X(f) is the Fourier transform of x(t).

- Matlab : cpsd, psd.

PSD using Matlab's pwelch

Assume x(t) is a signal sampled at fs (e.g. fs=1000 Hz).

To get the PSD:

[Pxx,frequency] = pwelch(x,window,noverlap,nfft,fs);

Where:

<u>window</u>: Window or window length. Specifying a number uses a Hamming window of the specified length (e.g. 1000 bins, a 1 second window)

<u>noverlap</u>: Overlap (in time) between calculation windows - how many time bins fall in both neighboring windows (e.g. 500 bins, 50% overlap)

<u>nfft</u>: FFT length - How many "points" in the resulting DFT.
Can set to window length, works faster with lengths that are powers of 2.
nfft = 2^(nextpow2('window length')).

Frequency bins will be fs(frquency range –Fs/2:Fs/2)/nfft (# points)

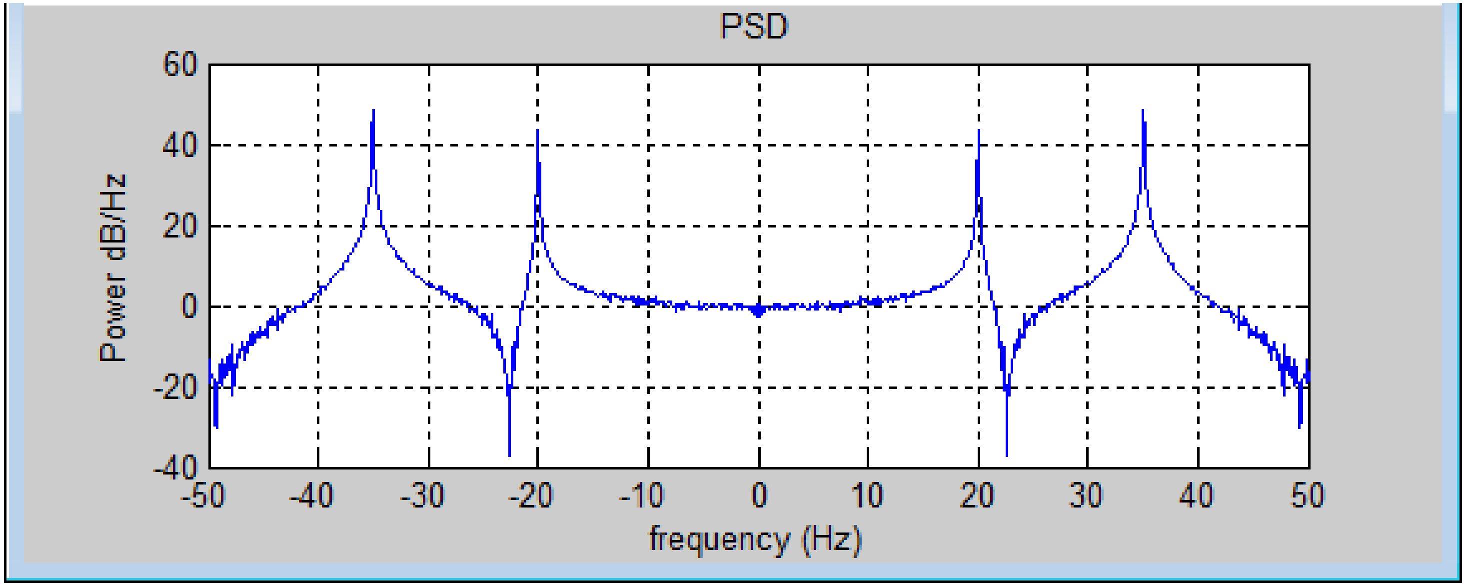
Example1: PSD with no overlap, bin of 1Hz: pwelch(x, fs, 0, fs, fs);

Example2: PSD with 50% overlap, bin of 0.5Hz: pwelch(x, 2*fs, fs, 2*fs, fs);

Example3: PSD 50% overlap, bin of 2Hz: pwelch(x, fs/2, fs/4, fs/2,fs);

Example PSD

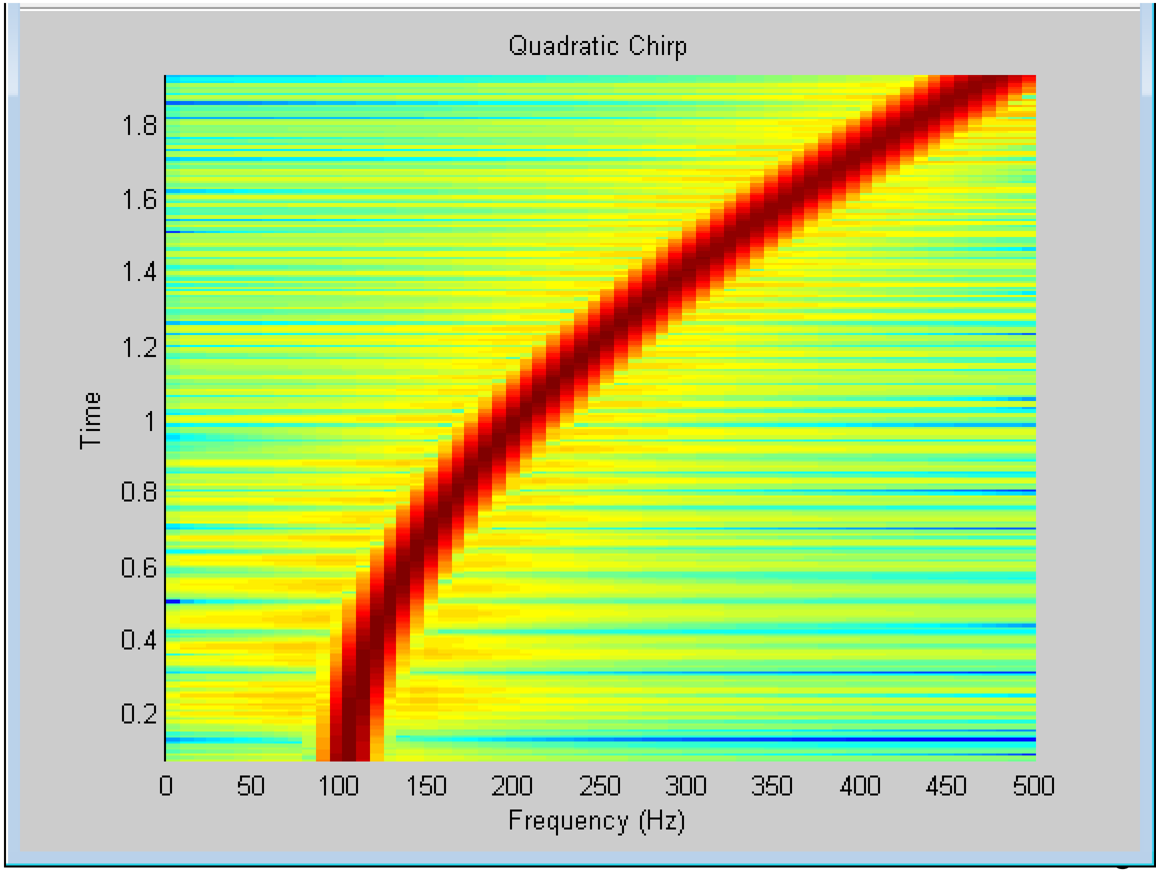
```
Fs = 100;          t = 0:1/Fs:10;
x = 100*cos(2*pi* 20*t)+ 200*cos(2*pi* 35*t)+ randn(size(t));
Pxx = abs(fftshift(fft(x))).^2/(length(x)*Fs);
freq = -Fs/2:Fs/(length(Pxx) -1):Fs/2;
plot(freq, 10*log10(Pxx));
xlabel('frequency (Hz)'); ylabel('Power dB/Hz'); title('PSD');
```



MATLAB EXAMPLE

Spectrogram

```
T = 0:0.001:2;
X = chirp(T,100,1,200,'q');
spectrogram(X,128,120,128,1000);   %(x,window,noverlap,nfft,fs)
title('Quadratic Chirp');
```



Coherence

- Points on spectral linear correlations.

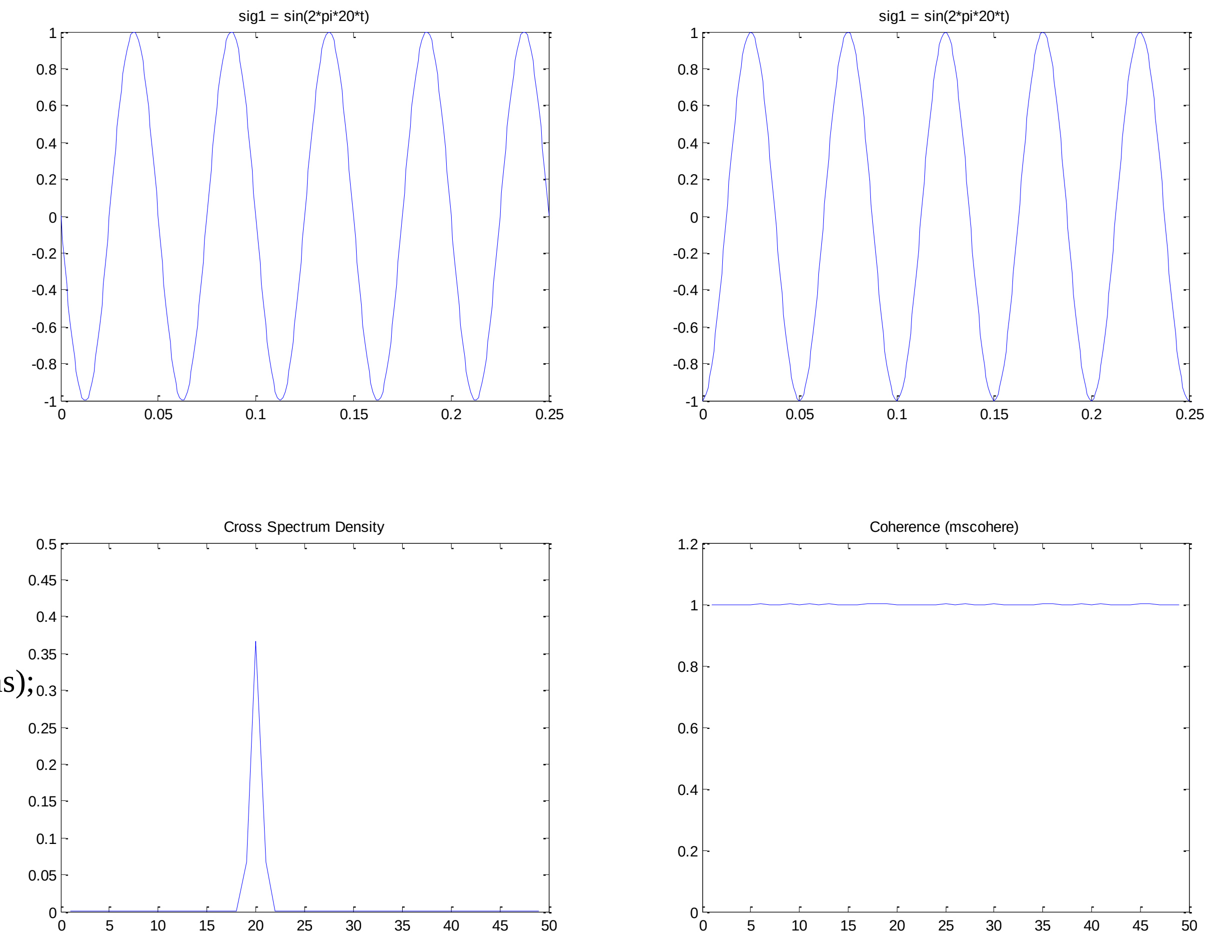
$$C_{xy} = \frac{|S_{xy}|^2}{S_{xx}S_{yy}}$$

$$0 \leq C_{xy} \leq 1$$

$$S \quad \text{spectral density}$$

Example

```
t = [0:0.001:100];
alpha = pi; beta = pi*1.5;
f1 = 20;
freqInds = 2:50;
pLims = [0 0.5];
sig1 = sin(2*pi*f1*t +alpha);
sig2 = sin(2*pi*f1*t+beta);
figure; subplot(2,2,1);
plot([0:0.001:0.25],sig1(1:251));
title(['sig1 = sin(2*pi*' num2str(f1) '*t)'])
subplot(2,2,2)
plot([0:0.001:0.25],sig2(1:251));
title(['sig1 = sin(2*pi*' num2str(f1) '*t)'])
[Pxy,F1] = cpsd(sig1,sig2,1000,0,1000,1000);
subplot(2,2,3)
plot(F1((freqInds)),abs(Pxy((freqInds)))); ylim(pLims);
title('Cross Spectrum Density')
[Cxy,F2] = mscohere(sig1,sig2,1000,0,1000,1000);
subplot(2,2,4)
plot(F2((freqInds)),Cxy((freqInds)));ylim([0 1.2]);
title('Coherence (mscohere)')
```



Exam 2005A: power spectrum

- The EEG of the Golden Unicorn is made up of three frequencies 7, 18 & 100Hz. The amplitude of each component (frequency) is 10μV. The waveform is sampled at 150 samples/sec for 1 second. Sketch and explain the power spectrum of the EEG. What are the X & Y axes? What is the X scale resolution? What is the maximal frequency?

Exam 2007: coherence

Given the two signals

$$a(t) = A \cdot \sin(20\pi t + \alpha) \qquad b(t) = B \cdot \sin(20\pi t + \beta)$$

The coherence between the two signals at 10Hz is:

a. The amplitude depends on A & B and the phase depends on α & β.

b. The amplitude does not depend on A & B and the phase depends on α & β.

c. The amplitude depends on A & B and the phase does not depend on α & β.

d. The amplitude does not depend on A & B and the phase does not depend on α & β.

e. None of the above.

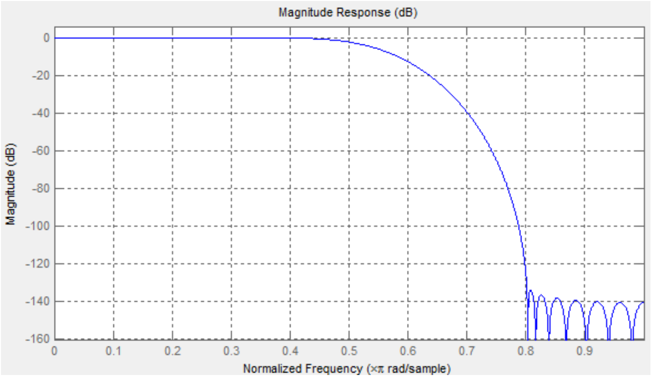
Filters

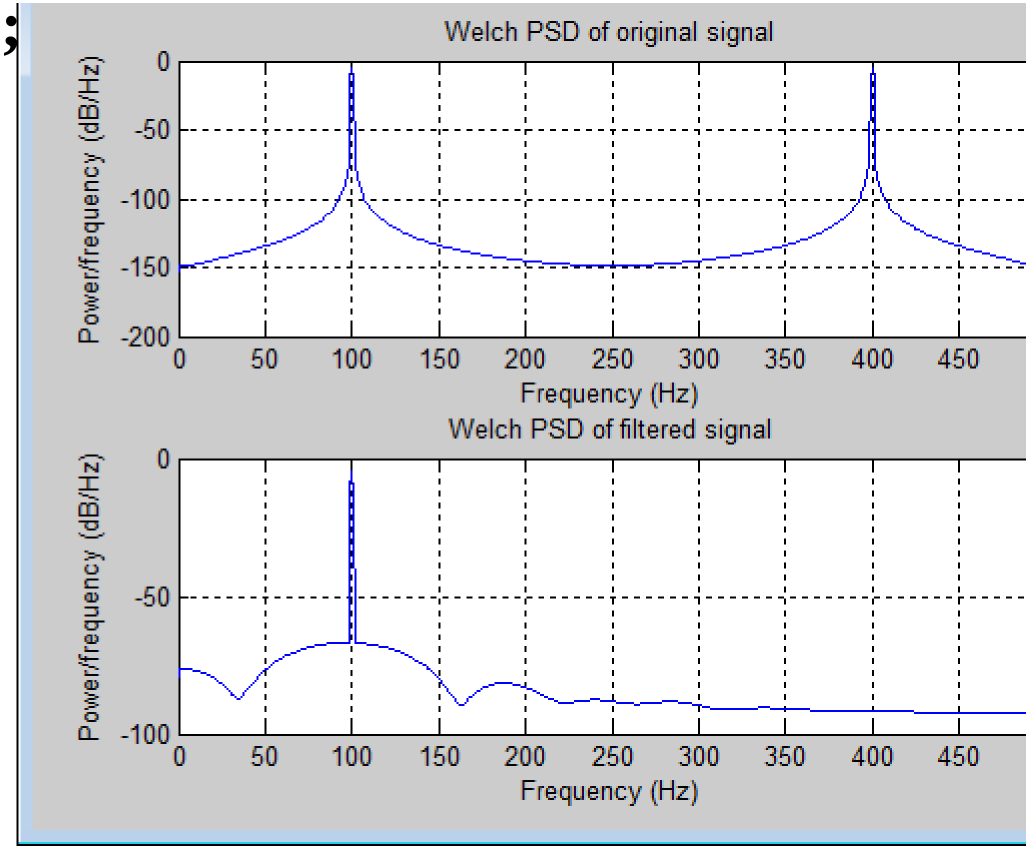
Filters

- A device or process that removes from a signal some unwanted component or feature.
- Filter classification:
  Spectral response: LPF, HPF, BPF, BSF, notch.
  - Digital filters:
    - FIR, IIR
    - Linear phase
- Options/tools for filter construction:
  - Matlab code
  - Matlab's filter visual tools (FDA tool, wintool)
  - Fvtool – Filter viewing tool

Filter design example - 1

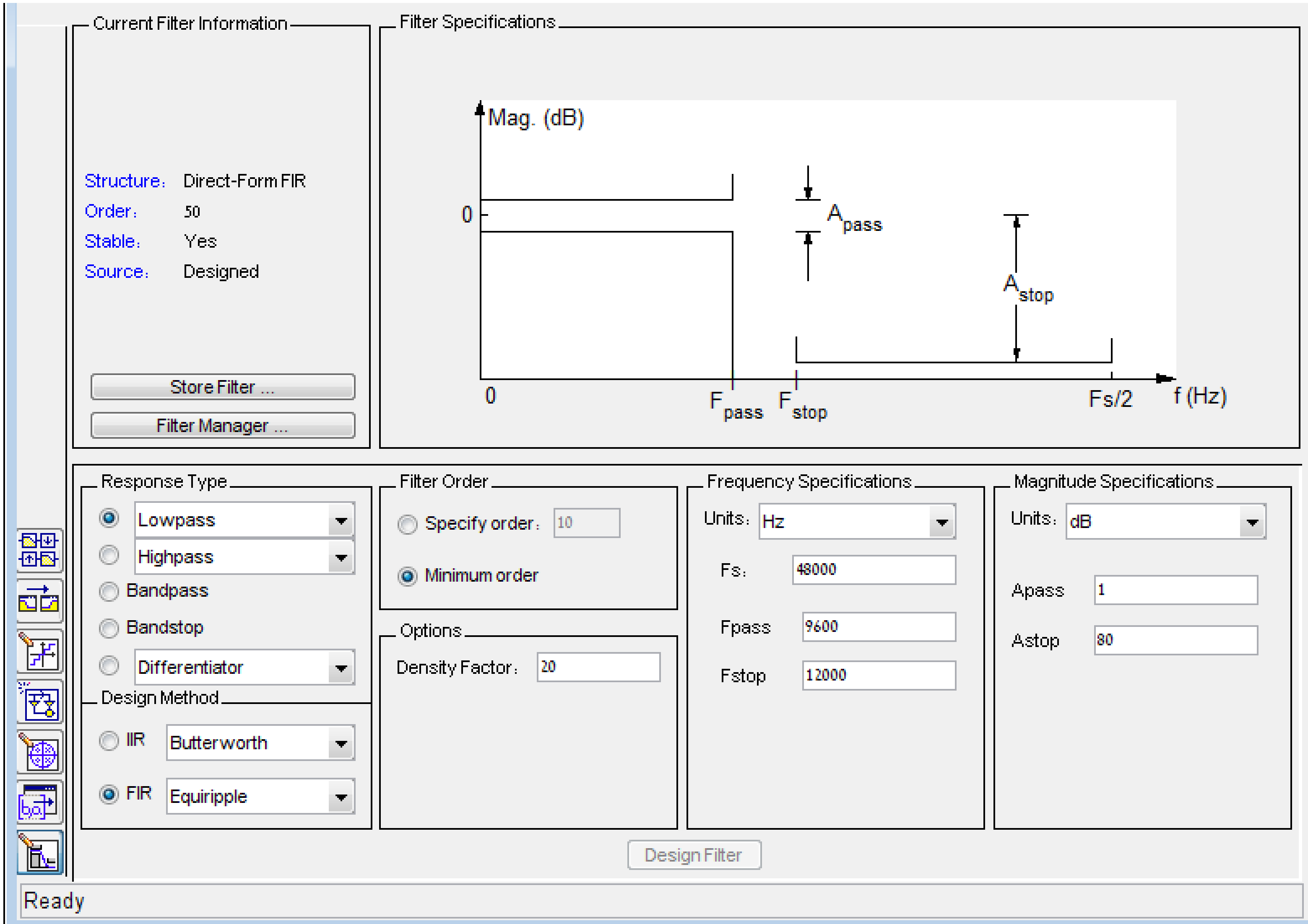
```
fs = 1e3;  t = 0:1/fs:10;
x = sin(2*pi*100*t) +sin(2*pi*400*t);
b = firls(32,[0 0.3 0.8 1],[1 1 0 0]);
y = filter(b, 1, x );
%[b,a] = iirnotch(w0, bw, m);
%y = filter(b, a, x );
% optional, filter response
fvtool(b, 1);
%filter signal and show PSD
subplot(2,1,1);
pwelch(x, fs, 0, fs, fs);
subplot(2,1,2);
pwelch(y, fs, 0, fs, fs);
```





Filter design example - 2

- Matlab: fdatool



Exam 2006: spectrum+ aliasing +filter

The signal $V(t) = X * \sin(20*t*2\pi) + Y * \cos(180*t*2\pi)$ is sampled at 100 samples/sec. The sampled signal is then filtered using a 40Hz perfect high pass filter. The power spectrum of the sampled signal displays the following:

a. Single peak at 20Hz.

b. Two peaks at 20Hz & 180Hz.

c. Single peak at 180Hz.

d. No peaks in the spectrum.

Exam 2007: Filters +FIR/IIR

. Two filters are given by the following equations:

1. $y(n) = x(n)-y(n-1)$
2. $y(n) =x(n)-x(n-1)$

a) Draw the impulse response of the two filters.

b) For each filter: is it FIR or IIR? Explain.

c) What is the output of the filters assuming a constant non-zero input? Explain

Exam 2005A: filters

Two filters are given by the following equations:

(1) $y(n)=2*y(n-1)+x(n)$

(2) $y(n)=8*x(n-3)+ 4*x(n-2)+ 2*x(n-1)+x(n)$

a) Draw the impulse response of the two filters.

b) For each filter: is it FIR or IIR? Explain.

c) What is the output of the filters assuming a constant non-zero input? Explain.